NAME: Solution

2.3 Model real-world phenomena using the fundamental definitions of relations.

The definition of the word ancestor is: "one from whom a person is descended," i.e., the parent of your parent, or their parent, or their parent, etc.

Define the following relation A for ancestors:

- For all people x and y we say $(x, y) \in A$ if x is an ancestor of y.

(a) Recall that an *inverse* of a relation R is a relation R^{-1} such that for all $(x, y) \in R \rightarrow (y, x) \in R^{-1}$. Provide a simple definition of the inverse of the ancestor relation A, and justify your definition in a couple sentences. (*Hint:* think about the definition of ancestor.)

We'll use the idea of a descendant. Define the relation $D = A^{-1}$ as follows. For all people x and y we say $(x, y) \in D$ if x is a descendant of y.

Note that if $(x, y) \in A$, then x is an ancestor of y. But that means y is descended from x, and so y is a descendant of x. Thus $(y, x) \in D$. So $D = A^{-1}$.

(b) A *strict partial ordering* is a relation that is irreflexive (not reflexive for any x), anti-symmetric, and transitive. Prove that the ancestor relation A is a strict partial ordering. A sentence or two for each property should suffice.

- Irreflexive: note that there is no person x who is there own ancestor, so $(x, x) \notin A$ for any x.

- Anti-symmetric: note that if x is an ancestor of y, then it is not possible for y to be an ancestor of x (you cannot be your own parent, grandparent, etc.). Thus if $(x, y) \in A$, then $(y, x) \notin A$.

- Transitive: If x is an ancestor of y and y is an ancestor of z, then it must be the case that z was descended from both y and x (through y). So x is also an ancestor of z. Thus if $(x, y) \in A$ and $(y, z) \in A$, then $(x, z) \in A$.

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