NAME: Solution

2.1 Author a rigorous proof of the equality of two sets.

Prove the following claim using set inclusion principles:

Claim. $(A - B) \cup B \equiv A \cup B$

(*Hint:* you'll need to show double containment ($\subseteq$)!)

We need to show double containment, so let's consider our two directions:

- $(A - B) \cup B \subseteq A \cup B$.

  Consider some $x \in (A - B) \cup B$. We know that $x \in A - B$ or $x \in B$. But, we know that $A - B \subseteq A$ since all elements in $A - B$ are in A. So we know that $x \in A$ or $x \in B$, which tells us that $x \in A \cup B$.

- $A \cup B \subseteq (A - B) \cup B$.

  Consider some $x \in A \cup B$. Then we know that $x \in A$ or $x \in B$. Now, if $x \in B$, then certainly $x \in (A - B) \cup B$. Let's consider what happens in $x \in A$ but is not also in B, i.e., if $x \in A$ but $x \notin B$. This means $x \in A - B$, and so again $x \in (A - B) \cup B$. Either way, $x \in (A - B) \cup B$.

We have shown double containment, and thus $(A - B) \cup B \equiv A \cup B$.

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